Supplementary Materials for "Superconducting micro-resonator arrays with ideal frequency spacing"

X. Liu[1], W. Guo[2,3*], Y. Wang[1], M. Dai[3], L. F. Wei[1,3,4], B. Dober[2], C. McKenney[2], G. C. Hilton[2], J. Hubmayr[2], J. E. Austermann[2], J. N. Ullom[2], J. Gao[2], and M. R. Vissers[2]

1) Quantum Optoelectronics Laboratory,

School of Physical Science and Technology,

Southwest Jiaotong University, Chengdu, 610031, China

2) National Institute of Standards and Technology, Boulder, CO 80305, USA[†]

3) Information Quantum Technology Laboratory,

School of Information Science and Technology,

Southwest Jiaotong University, Chengdu, 610031, China

4) State Key Laboratory of Optoelectronic Materials and Technologies,

School of Physics, Sun Yat-Sen University, Guangzhou 510275, China

(Dated: December 5, 2017)

[*] Author to whom correspondence should be addressed. Electronic mail: weijie.guo@nist.gov

Derivation of frequency collision probability and array yield formula

In this note, we assume an array of infinite number of resonators with the design resonance frequencies distributed in a geometric series, $f_n/f_{n-1} = 1+\Delta$ ($\Delta \ll 1$, and $\Delta = 0.002$ for the array in our paper). We pick an arbitrary resonator in the array (referred to as the 0^{th}-resonator) and assume there exist infinity number of resonators both above and below the frequency of the 0^{th} resonator f_0. This is an appropriate assumption for calculating the average yield for a large array of resonators. We model the actual resonance frequency of the n^{th} resonator as a random variable $\tilde{f}_n$ with a mean of $\langle \tilde{f}_n \rangle = f_n$ and assume its fractional frequency error (between the design and actual resonance frequency), $\frac{\tilde{f}_n - f_n}{f_n}$, follows a Gaussian distribution of $G(0, \sigma^2/2)$ which is independent for different n.

We define a new random variable $\tilde{x}_n$ by

$$\tilde{x_n} = \log(\frac{\tilde{f}_n}{\tilde{f}_0}) = \log(\tilde{f}_n) - \log(\tilde{f}_0), \tag{S1}$$

which transforms the frequency axis into a log-frequency axis and fixes $\tilde{x}_0$ at the origin (no longer a random variable). The advantage of this transformation is that $x_n = \langle \tilde{x}_n \rangle$ is now evenly spaced by Δ on the x-axis ($x_n = n\Delta$, see Fig. S1), which simplifies our following analysis. It follows that $\tilde{x}_n$ for each n also obeys an independent Gaussian distribution of $G(n\Delta, \sigma^2)$ (as shown in Fig. S1), where we have used the property that the difference of two independent Gaussian random variables is still a Gaussian random variable.

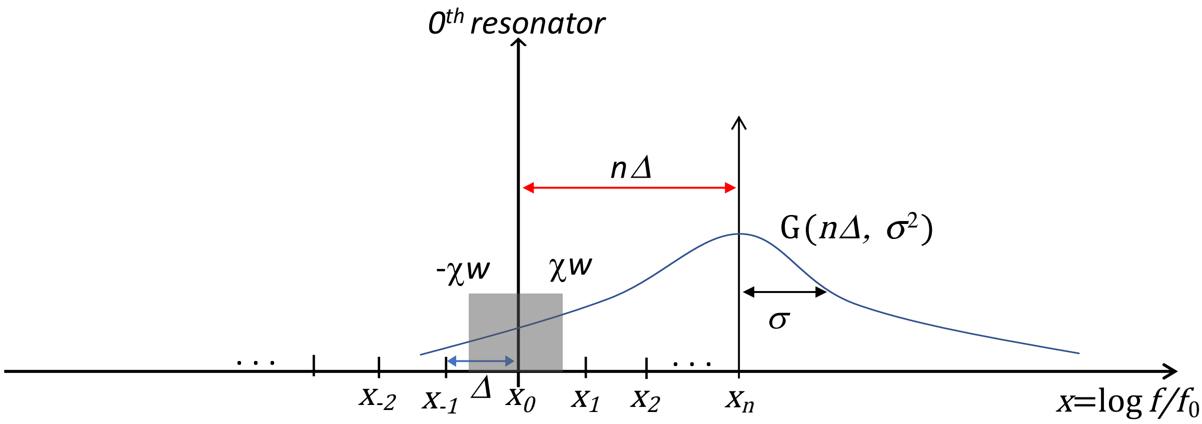

FIG. S1: Frequency collision model in log-frequency space.

We are now ready to calculate the probability P_{n0} for the 0^{th} resonator to survive the collision with the n^{th} resonator ($n = \pm 1, \pm 2, ..$). According to our χ-linewidth frequency collision criterion, collision between the n^{th} and 0^{th} resonators occurs when $\tilde{x}_n$ lies in the region of $[-\chi w, \chi w]$, where $w = 1/Q$ is the linewidth (unitless) of the resonator.

It follows that P_{n0} can be calculated in terms of an integral of the Gaussian distribution function in the region excluding $[-\chi w, \chi w]$,

$$\begin{aligned} P_{n0} &= 1 - \int_{-\chi w}^{+\chi w} G(x; n\Delta, \sigma^2) dx \\ &= 1 - \int_{\frac{-n\Delta - \chi w}{\sigma}}^{\frac{-n\Delta + \chi w}{\sigma}} G(x; 0, 1) dx \\ &= 1 - \frac{\mathrm{Erf}(\frac{n\Delta + \chi w}{\sqrt{2}\sigma}) - \mathrm{Erf}(\frac{n\Delta - \chi w)}{\sqrt{2}\sigma})}{2} \end{aligned} \tag{S2}$$

where Erf(x) is the error function defined as

$$\mathrm{Erf}(x) = \frac{1}{\sqrt{\pi}} \int_{-x}^{x} e^{-t^2} dt \tag{S3}$$

After getting the probability for the 0_{th} resonator to survive the collision with the n_{th} resonator, we can calculate the probability for the 0_{th} resonator to survive collisions with all the other resonators ($n = \pm 1, \pm 2, ...$) as

$$P_0 = \prod_n P_{n0} = \{\prod_{n=1}^{n=\infty} [1 - \frac{\mathrm{Erf}(\frac{n\Delta + \chi w}{\sqrt{2}\sigma}) - \mathrm{Erf}(\frac{n\Delta - \chi w}{\sqrt{2}\sigma})}{2}]\}^2 \tag{S4}$$

Eqn. S4 is the average yield of the array in terms of frequency collision.